

**Paper Specific Instructions**

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.
2. **Section – A** contains a total of **30 Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.
3. **Section – B** contains a total of **10 Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.
4. **Section – C** contains a total of **20 Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for this type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.
5. In all sections, questions not attempted will result in zero mark. In **Section – A (MCQ)**, wrong answer will result in **NEGATIVE** marks. For all 1 mark questions,  $\frac{1}{3}$  marks will be deducted for each wrong answer. For all 2 marks questions,  $\frac{2}{3}$  marks will be deducted for each wrong answer. In **Section – B (MSQ)**, there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C (NAT)** as well.
6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.
7. The Scribble Pad will be provided for rough work.

**NOTATION**

1.  $\mathbb{N} = \{1, 2, 3, \dots\}$
2.  $\mathbb{R}$  - the set of all real numbers
3.  $\mathbb{R} \setminus \{0\}$  - the set of all non-zero real numbers
4.  $\mathbb{C}$  - the set of all complex numbers
5.  $f \circ g$  - composition of the functions  $f$  and  $g$
6.  $f'$  and  $f''$  - first and second derivatives of the function  $f$ , respectively
7.  $f^{(n)}$  -  $n^{\text{th}}$  derivative of  $f$
8.  $\nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$
9.  $\oint_C$  - the line integral over an oriented closed curve  $C$
10.  $\hat{i}, \hat{j}, \hat{k}$  - unit vectors along the Cartesian right handed rectangular co-ordinate system
11.  $\hat{n}$  - unit outward normal vector
12.  $I$  - identity matrix of appropriate order
13.  $\det(M)$  - determinant of the matrix  $M$
14.  $M^{-1}$  - inverse of the matrix  $M$
15.  $M^T$  - transpose of the matrix  $M$
16.  $id$  - identity map
17.  $\langle a \rangle$  - cyclic subgroup generated by an element  $a$  of a group
18.  $S_n$  - permutation group on  $n$  symbols
19.  $S^1 = \{z \in \mathbb{C} : |z| = 1\}$
20.  $o(g)$  - order of the element  $g$  in a group

**SECTION – A**  
**MULTIPLE CHOICE QUESTIONS (MCQ)**

**Q. 1 – Q. 10 carry one mark each.**

Q. 1 Let  $s_n = 1 + \frac{(-1)^n}{n}$ ,  $n \in \mathbb{N}$ . Then the sequence  $\{s_n\}$  is

- (A) monotonically increasing and is convergent to 1
- (B) monotonically decreasing and is convergent to 1
- (C) neither monotonically increasing nor monotonically decreasing but is convergent to 1
- (D) divergent

Q. 2 Let  $f(x) = 2x^3 - 9x^2 + 7$ . Which of the following is true?

- (A)  $f$  is one-one in the interval  $[-1, 1]$
- (B)  $f$  is one-one in the interval  $[2, 4]$
- (C)  $f$  is NOT one-one in the interval  $[-4, 0]$
- (D)  $f$  is NOT one-one in the interval  $[0, 4]$

Q. 3 Which of the following is FALSE?

(A)  $\lim_{x \rightarrow \infty} \frac{x}{e^x} = 0$

(B)  $\lim_{x \rightarrow 0^+} \frac{1}{xe^{1/x}} = 0$

(C)  $\lim_{x \rightarrow 0^+} \frac{\sin x}{1 + 2x} = 0$

(D)  $\lim_{x \rightarrow 0^+} \frac{\cos x}{1 + 2x} = 0$

Q. 4 Let  $g : \mathbb{R} \rightarrow \mathbb{R}$  be a twice differentiable function. If  $f(x, y) = g(y) + xg'(y)$ , then

(A)  $\frac{\partial f}{\partial x} + y \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial y}$

(B)  $\frac{\partial f}{\partial y} + y \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial x}$

(C)  $\frac{\partial f}{\partial x} + x \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial y}$

(D)  $\frac{\partial f}{\partial y} + x \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial x}$

Q. 5 If the equation of the tangent plane to the surface  $z = 16 - x^2 - y^2$  at the point  $P(1, 3, 6)$  is  $ax + by + cz + d = 0$ , then the value of  $|d|$  is

- (A) 16                      (B) 26                      (C) 36                      (D) 46

Q. 6 If the directional derivative of the function  $z = y^2 e^{2x}$  at  $(2, -1)$  along the unit vector  $\vec{b} = \alpha \hat{i} + \beta \hat{j}$  is zero, then  $|\alpha + \beta|$  equals

- (A)  $\frac{1}{2\sqrt{2}}$                       (B)  $\frac{1}{\sqrt{2}}$                       (C)  $\sqrt{2}$                       (D)  $2\sqrt{2}$

Q. 7 If  $u = x^3$  and  $v = y^2$  transform the differential equation  $3x^5 dx - y(y^2 - x^3)dy = 0$  to  $\frac{dv}{du} = \frac{\alpha u}{2(u - v)}$ , then  $\alpha$  is

- (A) 4                      (B) 2                      (C) -2                      (D) -4

Q. 8 Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear transformation given by  $T(x, y) = (-x, y)$ . Then

- (A)  $T^{2k} = T$  for all  $k \geq 1$   
(B)  $T^{2k+1} = -T$  for all  $k \geq 1$   
(C) the range of  $T^2$  is a proper subspace of the range of  $T$   
(D) the range of  $T^2$  is equal to the range of  $T$

Q. 9 The radius of convergence of the power series

$$\sum_{n=1}^{\infty} \left(\frac{n+2}{n}\right)^{n^2} x^n$$

is

- (A)  $e^2$                       (B)  $\frac{1}{\sqrt{e}}$                       (C)  $\frac{1}{e}$                       (D)  $\frac{1}{e^2}$

Q. 10 Consider the following group under matrix multiplication:

$$H = \left\{ \begin{bmatrix} 1 & p & q \\ 0 & 1 & r \\ 0 & 0 & 1 \end{bmatrix} : p, q, r \in \mathbb{R} \right\}.$$

Then the center of the group is isomorphic to

- (A)  $(\mathbb{R} \setminus \{0\}, \times)$  (B)  $(\mathbb{R}, +)$   
 (C)  $(\mathbb{R}^2, +)$  (D)  $(\mathbb{R}, +) \times (\mathbb{R} \setminus \{0\}, \times)$

**Q. 11 – Q. 30 carry two marks each.**

Q. 11 Let  $\{a_n\}$  be a sequence of positive real numbers. Suppose that  $l = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$ . Which of the following is true?

- (A) If  $l = 1$ , then  $\lim_{n \rightarrow \infty} a_n = 1$  (B) If  $l = 1$ , then  $\lim_{n \rightarrow \infty} a_n = 0$   
 (C) If  $l < 1$ , then  $\lim_{n \rightarrow \infty} a_n = 1$  (D) If  $l < 1$ , then  $\lim_{n \rightarrow \infty} a_n = 0$

Q. 12 Define  $s_1 = \alpha > 0$  and  $s_{n+1} = \sqrt{\frac{1 + s_n^2}{1 + \alpha}}$ ,  $n \geq 1$ . Which of the following is true?

- (A) If  $s_n^2 < \frac{1}{\alpha}$ , then  $\{s_n\}$  is monotonically increasing and  $\lim_{n \rightarrow \infty} s_n = \frac{1}{\sqrt{\alpha}}$   
 (B) If  $s_n^2 < \frac{1}{\alpha}$ , then  $\{s_n\}$  is monotonically decreasing and  $\lim_{n \rightarrow \infty} s_n = \frac{1}{\alpha}$   
 (C) If  $s_n^2 > \frac{1}{\alpha}$ , then  $\{s_n\}$  is monotonically increasing and  $\lim_{n \rightarrow \infty} s_n = \frac{1}{\sqrt{\alpha}}$   
 (D) If  $s_n^2 > \frac{1}{\alpha}$ , then  $\{s_n\}$  is monotonically decreasing and  $\lim_{n \rightarrow \infty} s_n = \frac{1}{\alpha}$

Q. 13 Suppose that  $S$  is the sum of a convergent series  $\sum_{n=1}^{\infty} a_n$ . Define  $t_n = a_n + a_{n+1} + a_{n+2}$ . Then

the series  $\sum_{n=1}^{\infty} t_n$

- (A) diverges  
 (B) converges to  $3S - a_1 - a_2$   
 (C) converges to  $3S - a_1 - 2a_2$   
 (D) converges to  $3S - 2a_1 - a_2$

Q. 14 Let  $a \in \mathbb{R}$ . If  $f(x) = \begin{cases} (x+a)^2, & x \leq 0 \\ (x+a)^3, & x > 0, \end{cases}$

then

- (A)  $\frac{d^2f}{dx^2}$  does not exist at  $x = 0$  for any value of  $a$   
 (B)  $\frac{d^2f}{dx^2}$  exists at  $x = 0$  for exactly one value of  $a$   
 (C)  $\frac{d^2f}{dx^2}$  exists at  $x = 0$  for exactly two values of  $a$   
 (D)  $\frac{d^2f}{dx^2}$  exists at  $x = 0$  for infinitely many values of  $a$

Q. 15 Let  $f(x, y) = \begin{cases} x^2 \sin \frac{1}{x} + y^2 \sin \frac{1}{y}, & xy \neq 0 \\ x^2 \sin \frac{1}{x}, & x \neq 0, y = 0 \\ y^2 \sin \frac{1}{y}, & y \neq 0, x = 0 \\ 0, & x = y = 0. \end{cases}$

Which of the following is true at  $(0, 0)$ ?

- (A)  $f$  is not continuous  
 (B)  $\frac{\partial f}{\partial x}$  is continuous but  $\frac{\partial f}{\partial y}$  is not continuous  
 (C)  $f$  is not differentiable  
 (D)  $f$  is differentiable but both  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  are not continuous

- Q. 16 Let  $S$  be the surface of the portion of the sphere with centre at the origin and radius 4, above the  $xy$ -plane. Let  $\vec{F} = y\hat{i} - x\hat{j} + yx^3\hat{k}$ . If  $\hat{n}$  is the unit outward normal to  $S$ , then

$$\iint_S (\nabla \times \vec{F}) \cdot \hat{n} \, dS$$

equals

- (A)  $-32\pi$                       (B)  $-16\pi$                       (C)  $16\pi$                       (D)  $32\pi$
- Q. 17 Let  $f(x, y, z) = x^3 + y^3 + z^3 - 3xyz$ . A point at which the gradient of the function  $f$  is equal to zero is
- (A)  $(-1, 1, -1)$               (B)  $(-1, -1, -1)$               (C)  $(-1, 1, 1)$               (D)  $(1, -1, 1)$

- Q. 18 The area bounded by the curves  $x^2 + y^2 = 2x$  and  $x^2 + y^2 = 4x$ , and the straight lines  $y = x$  and  $y = 0$  is

(A)  $3\left(\frac{\pi}{2} + \frac{1}{4}\right)$               (B)  $3\left(\frac{\pi}{4} + \frac{1}{2}\right)$               (C)  $2\left(\frac{\pi}{4} + \frac{1}{3}\right)$               (D)  $2\left(\frac{\pi}{3} + \frac{1}{4}\right)$

- Q. 19 Let  $M$  be a real  $6 \times 6$  matrix. Let 2 and  $-1$  be two eigenvalues of  $M$ . If  $M^5 = aI + bM$ , where  $a, b \in \mathbb{R}$ , then

- (A)  $a = 10, b = 11$                       (B)  $a = -11, b = 10$   
(C)  $a = -10, b = 11$                       (D)  $a = 10, b = -11$

- Q. 20 Let  $M$  be an  $n \times n$  ( $n \geq 2$ ) non-zero real matrix with  $M^2 = 0$  and let  $\alpha \in \mathbb{R} \setminus \{0\}$ . Then

- (A)  $\alpha$  is the only eigenvalue of  $(M + \alpha I)$  and  $(M - \alpha I)$   
(B)  $\alpha$  is the only eigenvalue of  $(M + \alpha I)$  and  $(\alpha I - M)$   
(C)  $-\alpha$  is the only eigenvalue of  $(M + \alpha I)$  and  $(M - \alpha I)$   
(D)  $-\alpha$  is the only eigenvalue of  $(M + \alpha I)$  and  $(\alpha I - M)$

Q. 21 Consider the differential equation  $L[y] = (y - y^2)dx + xdy = 0$ . The function  $f(x, y)$  is said to be an integrating factor of the equation if  $f(x, y)L[y] = 0$  becomes exact.

If  $f(x, y) = \frac{1}{x^2y^2}$ , then

- (A)  $f$  is an integrating factor and  $y = 1 - kxy$ ,  $k \in \mathbb{R}$  is NOT its general solution
- (B)  $f$  is an integrating factor and  $y = -1 + kxy$ ,  $k \in \mathbb{R}$  is its general solution
- (C)  $f$  is an integrating factor and  $y = -1 + kxy$ ,  $k \in \mathbb{R}$  is NOT its general solution
- (D)  $f$  is NOT an integrating factor and  $y = 1 + kxy$ ,  $k \in \mathbb{R}$  is its general solution

Q. 22 A solution of the differential equation  $2x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - y = 0$ ,  $x > 0$  that passes through the point  $(1, 1)$  is

- (A)  $y = \frac{1}{x}$
- (B)  $y = \frac{1}{x^2}$
- (C)  $y = \frac{1}{\sqrt{x}}$
- (D)  $y = \frac{1}{x^{3/2}}$

Q. 23 Let  $M$  be a  $4 \times 3$  real matrix and let  $\{e_1, e_2, e_3\}$  be the standard basis of  $\mathbb{R}^3$ . Which of the following is true?

- (A) If  $\text{rank}(M) = 1$ , then  $\{Me_1, Me_2\}$  is a linearly independent set
- (B) If  $\text{rank}(M) = 2$ , then  $\{Me_1, Me_2\}$  is a linearly independent set
- (C) If  $\text{rank}(M) = 2$ , then  $\{Me_1, Me_3\}$  is a linearly independent set
- (D) If  $\text{rank}(M) = 3$ , then  $\{Me_1, Me_3\}$  is a linearly independent set

Q. 24 The value of the triple integral  $\iiint_V (x^2y + 1) dx dy dz$ , where  $V$  is the region given by  $x^2 + y^2 \leq 1, 0 \leq z \leq 2$  is

- (A)  $\pi$
- (B)  $2\pi$
- (C)  $3\pi$
- (D)  $4\pi$

Q. 25 Let  $S$  be the part of the cone  $z^2 = x^2 + y^2$  between the planes  $z = 0$  and  $z = 1$ . Then the value of the surface integral  $\iint_S (x^2 + y^2) dS$  is

- (A)  $\pi$
- (B)  $\frac{\pi}{\sqrt{2}}$
- (C)  $\frac{\pi}{\sqrt{3}}$
- (D)  $\frac{\pi}{2}$



Q. 26 Let  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ ,  $x, y, z \in \mathbb{R}$ . Which of the following is FALSE?

- (A)  $\nabla(\vec{a} \cdot \vec{r}) = \vec{a}$  (B)  $\nabla \cdot (\vec{a} \times \vec{r}) = 0$   
 (C)  $\nabla \times (\vec{a} \times \vec{r}) = \vec{a}$  (D)  $\nabla \cdot ((\vec{a} \cdot \vec{r})\vec{r}) = 4(\vec{a} \cdot \vec{r})$

Q. 27 Let  $D = \{(x, y) \in \mathbb{R}^2 : |x| + |y| \leq 1\}$  and  $f : D \rightarrow \mathbb{R}$  be a non-constant continuous function. Which of the following is TRUE?

- (A) The range of  $f$  is unbounded  
 (B) The range of  $f$  is a union of open intervals  
 (C) The range of  $f$  is a closed interval  
 (D) The range of  $f$  is a union of at least two disjoint closed intervals

Q. 28 Let  $f : [0, 1] \rightarrow \mathbb{R}$  be a continuous function such that  $f\left(\frac{1}{2}\right) = -\frac{1}{2}$  and

$$|f(x) - f(y) - (x - y)| \leq \sin(|x - y|^2)$$

for all  $x, y \in [0, 1]$ . Then  $\int_0^1 f(x) dx$  is

- (A)  $-\frac{1}{2}$  (B)  $-\frac{1}{4}$  (C)  $\frac{1}{4}$  (D)  $\frac{1}{2}$

Q. 29 Let  $S^1 = \{z \in \mathbb{C} : |z| = 1\}$  be the circle group under multiplication and  $i = \sqrt{-1}$ . Then the set  $\{\theta \in \mathbb{R} : \langle e^{i2\pi\theta} \rangle \text{ is infinite}\}$  is

- (A) empty (B) non-empty and finite  
 (C) countably infinite (D) uncountable

Q. 30 Let  $F = \{\omega \in \mathbb{C} : \omega^{2020} = 1\}$ . Consider the groups

$$G = \left\{ \begin{pmatrix} \omega & z \\ 0 & 1 \end{pmatrix} : \omega \in F, z \in \mathbb{C} \right\}$$

and

$$H = \left\{ \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix} : z \in \mathbb{C} \right\}$$

under matrix multiplication. Then the number of cosets of  $H$  in  $G$  is

(A) 1010

(B) 2019

(C) 2020

(D) infinite

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**SECTION – B**  
**MULTIPLE SELECT QUESTIONS (MSQ)**

**Q. 31 – Q. 40 carry two marks each.**

Q. 31 Let  $a, b, c \in \mathbb{R}$  such that  $a < b < c$ . Which of the following is/are true for any continuous function  $f : \mathbb{R} \rightarrow \mathbb{R}$  satisfying  $f(a) = b, f(b) = c$  and  $f(c) = a$ ?

- (A) There exists  $\alpha \in (a, c)$  such that  $f(\alpha) = \alpha$
- (B) There exists  $\beta \in (a, b)$  such that  $f(\beta) = \beta$
- (C) There exists  $\gamma \in (a, b)$  such that  $(f \circ f)(\gamma) = \gamma$
- (D) There exists  $\delta \in (a, c)$  such that  $(f \circ f \circ f)(\delta) = \delta$

Q. 32 If  $s_n = \frac{(-1)^n}{2n+3}$  and  $t_n = \frac{(-1)^n}{4n-1}, n = 0, 1, 2, \dots$ , then

- (A)  $\sum_{n=0}^{\infty} s_n$  is absolutely convergent
- (B)  $\sum_{n=0}^{\infty} t_n$  is absolutely convergent
- (C)  $\sum_{n=0}^{\infty} s_n$  is conditionally convergent
- (D)  $\sum_{n=0}^{\infty} t_n$  is conditionally convergent

Q. 33 Let  $a, b \in \mathbb{R}$  and  $a < b$ . Which of the following statement(s) is/are true?

- (A) There exists a continuous function  $f : [a, b] \rightarrow (a, b)$  such that  $f$  is one-one
- (B) There exists a continuous function  $f : [a, b] \rightarrow (a, b)$  such that  $f$  is onto
- (C) There exists a continuous function  $f : (a, b) \rightarrow [a, b]$  such that  $f$  is one-one
- (D) There exists a continuous function  $f : (a, b) \rightarrow [a, b]$  such that  $f$  is onto

Q. 34 Let  $V$  be a non-zero vector space over a field  $F$ . Let  $S \subset V$  be a non-empty set. Consider the following properties of  $S$ :

(I) For any vector space  $W$  over  $F$ , any map  $f : S \rightarrow W$  extends to a linear map from  $V$  to  $W$ .

(II) For any vector space  $W$  over  $F$  and any two linear maps  $f, g : V \rightarrow W$  satisfying  $f(s) = g(s)$  for all  $s \in S$ , we have  $f(v) = g(v)$  for all  $v \in V$ .

(III)  $S$  is linearly independent.

(IV) The span of  $S$  is  $V$ .

Which of the following statement(s) is /are true?

(A) (I) implies (IV)

(B) (I) implies (III)

(C) (II) implies (III)

(D) (II) implies (IV)

Q. 35 Let  $L[y] = x^2 \frac{d^2 y}{dx^2} + px \frac{dy}{dx} + qy$ , where  $p, q$  are real constants. Let  $y_1(x)$  and  $y_2(x)$  be two solutions of  $L[y] = 0, x > 0$ , that satisfy  $y_1(x_0) = 1, y_1'(x_0) = 0, y_2(x_0) = 0$  and  $y_2'(x_0) = 1$  for some  $x_0 > 0$ . Then,

(A)  $y_1(x)$  is not a constant multiple of  $y_2(x)$

(B)  $y_1(x)$  is a constant multiple of  $y_2(x)$

(C)  $1, \ln x$  are solutions of  $L[y] = 0$  when  $p = 1, q = 0$

(D)  $x, \ln x$  are solutions of  $L[y] = 0$  when  $p + q \neq 0$

Q. 36 Consider the following system of linear equations

$$x + y + 5z = 3, \quad x + 2y + mz = 5 \quad \text{and} \quad x + 2y + 4z = k.$$

The system is consistent if

(A)  $m \neq 4$

(B)  $k \neq 5$

(C)  $m = 4$

(D)  $k = 5$

Q. 37 Let  $a = \lim_{n \rightarrow \infty} \left( \frac{1}{n^2} + \frac{2}{n^2} + \cdots + \frac{(n-1)}{n^2} \right)$  and  $b = \lim_{n \rightarrow \infty} \left( \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n} \right)$ .

Which of the following is/are true?

- (A)  $a > b$                       (B)  $a < b$                       (C)  $ab = \ln \sqrt{2}$                       (D)  $\frac{a}{b} = \ln \sqrt{2}$

Q. 38 Let  $S$  be that part of the surface of the paraboloid  $z = 16 - x^2 - y^2$  which is above the plane  $z = 0$  and  $D$  be its projection on the  $xy$ -plane. Then the area of  $S$  equals

- (A)  $\iint_D \sqrt{1 + 4(x^2 + y^2)} \, dx dy$                       (B)  $\iint_D \sqrt{1 + 2(x^2 + y^2)} \, dx dy$   
 (C)  $\int_0^{2\pi} \int_0^4 \sqrt{1 + 4r^2} \, r dr d\theta$                       (D)  $\int_0^{2\pi} \int_0^4 \sqrt{1 + 4r^2} \, r dr d\theta$

Q. 39 Let  $f$  be a real valued function of a real variable, such that  $|f^{(n)}(0)| \leq K$  for all  $n \in \mathbb{N}$ , where  $K > 0$ . Which of the following is/are true?

- (A)  $\left| \frac{f^{(n)}(0)}{n!} \right|^{\frac{1}{n}} \rightarrow 0$  as  $n \rightarrow \infty$   
 (B)  $\left| \frac{f^{(n)}(0)}{n!} \right|^{\frac{1}{n}} \rightarrow \infty$  as  $n \rightarrow \infty$   
 (C)  $f^{(n)}(x)$  exists for all  $x \in \mathbb{R}$  and for all  $n \in \mathbb{N}$   
 (D) The series  $\sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{(n-1)!}$  is absolutely convergent

Q. 40 Let  $G$  be a group with identity  $e$ . Let  $H$  be an abelian non-trivial proper subgroup of  $G$  with the property that  $H \cap gHg^{-1} = \{e\}$  for all  $g \notin H$ .

If  $K = \{g \in G : gh = hg \text{ for all } h \in H\}$ , then

- (A)  $K$  is a proper subgroup of  $H$   
 (B)  $H$  is a proper subgroup of  $K$   
 (C)  $K = H$   
 (D) there exists no abelian subgroup  $L \subseteq G$  such that  $K$  is a proper subgroup of  $L$

**SECTION – C**  
**NUMERICAL ANSWER TYPE (NAT)**

**Q. 41 – Q. 50 carry one mark each.**

Q. 41 Let  $x_n = n^{\frac{1}{n}}$  and  $y_n = e^{1-x_n}$ ,  $n \in \mathbb{N}$ . Then the value of  $\lim_{n \rightarrow \infty} y_n$  is \_\_\_\_\_.

Q. 42 Let  $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$  and  $S$  be the sphere given by  $(x - 2)^2 + (y - 2)^2 + (z - 2)^2 = 4$ . If  $\hat{n}$  is the unit outward normal to  $S$ , then

$$\frac{1}{\pi} \iint_S \vec{F} \cdot \hat{n} \, dS$$

is \_\_\_\_\_.

Q. 43 Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be such that  $f, f', f''$  are continuous functions with  $f > 0, f' > 0$  and  $f'' > 0$ . Then

$$\lim_{x \rightarrow -\infty} \frac{f(x) + f'(x)}{2}$$

is \_\_\_\_\_.

Q. 44 Let  $S = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$  and  $f : S \rightarrow \mathbb{R}$  be defined by  $f(x) = \frac{1}{x}$ . Then

$$\max \left\{ \delta : \left| x - \frac{1}{3} \right| < \delta \implies \left| f(x) - f\left(\frac{1}{3}\right) \right| < 1 \right\}$$

is \_\_\_\_\_. (rounded off to two decimal places)

Q. 45 Let  $f(x, y) = e^x \sin y$ ,  $x = t^3 + 1$  and  $y = t^4 + t$ . Then  $\frac{df}{dt}$  at  $t = 0$  is \_\_\_\_\_. (rounded off to two decimal places)

Q. 46 Consider the differential equation

$$\frac{dy}{dx} + 10y = f(x), \quad x > 0,$$

where  $f(x)$  is a continuous function such that  $\lim_{x \rightarrow \infty} f(x) = 1$ . Then the value of

$$\lim_{x \rightarrow \infty} y(x)$$

is \_\_\_\_\_.

Q. 47 If  $\int_0^1 \int_{2y}^2 e^{x^2} dx dy = k(e^4 - 1)$ , then  $k$  equals \_\_\_\_\_.

Q. 48 Let  $f(x, y) = 0$  be a solution of the homogeneous differential equation

$$(2x + 5y)dx - (x + 3y)dy = 0.$$

If  $f(x + \alpha, y - 3) = 0$  is a solution of the differential equation

$$(2x + 5y - 1)dx + (2 - x - 3y)dy = 0,$$

then the value of  $\alpha$  is \_\_\_\_\_.

Q. 49 Consider the real vector space  $P_{2020} = \left\{ \sum_{i=0}^n a_i x^i : a_i \in \mathbb{R} \text{ and } 0 \leq n \leq 2020 \right\}$ . Let  $W$  be the subspace given by

$$W = \left\{ \sum_{i=0}^n a_i x^i \in P_{2020} : a_i = 0 \text{ for all odd } i \right\}.$$

Then, the dimension of  $W$  is \_\_\_\_\_.

Q. 50 Let  $\phi : S_3 \rightarrow S^1$  be a non-trivial non-injective group homomorphism. Then, the number of elements in the kernel of  $\phi$  is \_\_\_\_\_.

**Q. 51 – Q. 60 carry two marks each.**

Q. 51 The sum of the series  $\frac{1}{2(2^2 - 1)} + \frac{1}{3(3^2 - 1)} + \frac{1}{4(4^2 - 1)} + \dots$  is \_\_\_\_\_.

Q. 52 Consider the expansion of the function  $f(x) = \frac{3}{(1-x)(1+2x)}$  in powers of  $x$ , that is valid in  $|x| < \frac{1}{2}$ . Then the coefficient of  $x^4$  is \_\_\_\_\_.

Q. 53 The minimum value of the function  $f(x, y) = x^2 + xy + y^2 - 3x - 6y + 11$  is \_\_\_\_\_.

Q. 54 Let  $f(x) = \sqrt{x} + \alpha x$ ,  $x > 0$  and

$$g(x) = a_0 + a_1(x-1) + a_2(x-1)^2$$

be the sum of the first three terms of the Taylor series of  $f(x)$  around  $x = 1$ . If  $g(3) = 3$ , then  $\alpha$  is \_\_\_\_\_.

Q. 55 Let  $C$  be the boundary of the square with vertices  $(0, 0)$ ,  $(1, 0)$ ,  $(1, 1)$  and  $(0, 1)$  oriented in the counter clockwise sense. Then, the value of the line integral

$$\oint_C x^2 y^2 dx + (x^2 - y^2) dy$$

is \_\_\_\_\_. (rounded off to two decimal places)

Q. 56 Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function with  $f'(x) = f(x)$  for all  $x$ . Suppose that  $f(\alpha x)$  and  $f(\beta x)$  are two non-zero solutions of the differential equation

$$4 \frac{d^2 y}{dx^2} - p \frac{dy}{dx} + 3y = 0$$

satisfying

$$f(\alpha x)f(\beta x) = f(2x) \quad \text{and} \quad f(\alpha x)f(-\beta x) = f(x).$$

Then, the value of  $p$  is \_\_\_\_\_.



Q. 57 If  $x^2 + xy^2 = c$ , where  $c \in \mathbb{R}$ , is the general solution of the exact differential equation

$$M(x, y) dx + 2xy dy = 0,$$

then  $M(1, 1)$  is \_\_\_\_\_.

Q. 58 Let  $M = \begin{bmatrix} 9 & 2 & 7 & 1 \\ 0 & 7 & 2 & 1 \\ 0 & 0 & 11 & 6 \\ 0 & 0 & -5 & 0 \end{bmatrix}$ . Then, the value of  $\det((8I - M)^3)$  is \_\_\_\_\_.

Q. 59 Let  $T : \mathbb{R}^7 \rightarrow \mathbb{R}^7$  be a linear transformation with  $\text{Nullity}(T) = 2$ . Then, the minimum possible value for  $\text{Rank}(T^2)$  is \_\_\_\_\_.

Q. 60 Suppose that  $G$  is a group of order 57 which is NOT cyclic. If  $G$  contains a unique subgroup  $H$  of order 19, then for any  $g \notin H$ ,  $o(g)$  is \_\_\_\_\_.

**END OF THE QUESTION PAPER**